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Kaiser, Uwe

Verschlingungsabbildungen im euklidischen Raum. (Link maps into Euclidean space). (German)

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A (p,q)-link map into a space X is a map f from $S^p \cup S^q$ to X such that $f(S^p)$ and $f(S^q)$ are disjoint, and a link homotopy of (p,q)-link maps is a homotopy through such maps. The objects of this dissertation are to compare the sets $ELM^m_{p,q}$ and $LM^m_{p,q}$ of link homotopy classes of (p,q)-link maps into R^m and S^m, respectively, and to find invariants to distinguish between members of these sets. (We shall henceforth suppose that $q \leq p$ and suppress the indices). The inclusion of R^m into $S^m = R^m \cup \{\infty\}$ induces a map $incl_*$ from ELM to LM which is onto if $m \geq 1$ and is bijective if also $p \leq m-2$. However, if $p \geq m-1$ the relationship of these sets is not so simple. The most general result obtained here is that if $q \leq m-3$ and $p+2q \leq 3m-6$ then there is a bijection from ELM to $LM \times \pi_p(S^{m-1}) \times \pi_{p+q}(S^{2m-3})$ (Theorem 13.17). Outside this range of dimensions less is known, although a similar result holds for homotopy classes of the (p,q)-link maps obtainable from $(p-1,q)-link$ maps into R^{m-1} (respectively, S^{m-1}) by "suspension", provided only that $q \leq m-3$ (Theorem 10.6).

Three maps play important roles in this work. Firstly, there is a map b from LM to ELM such that $incl_* b = id_{LM}$ (Section 2). If $1 \leq q \leq m-3$ the set LM is a monoid with respect to connected sum, and acts on ELM in a way compatible with the maps $incl_*$ and b. The difference map ϕ from ELM to $[S^p \times S^q, S^{m-1}]$ sends the class represented by a map with components f_p and f_q to the homotopy class of the map sending (x,y) in $S^p \times S^q$ to $(f_p(x) - f_q(y))/|f_p(x) - f_q(x)|$. (This is introduced in Section 4, composed with the Hopf homomorhism in Section 6 and generalised in Sections 8 and 9). If $q \leq m-1$ there is a map e_* from $\pi_p(S^{m-q-1}\nu S^{m-1})$ to ELM whose image consists of classes which have a representative (p,q)-link map f such that $f|S^q$ is an unknotted embedding, and which is bijective if $2q+2 \leq 2$ (Proposition 5.12). Using e_* we may find nontrivial classes in ELM which become trivial in LM. Sections 10 and 11 consider criteria for a link map to be "spherical" (in the image of b) and derive obstructions to sphericity. Section 12 computes certain of these invariants on the image of e_*. They are investigated further via an interpretation of certain homotopy sets as bordism of framed submanifolds of Euclidean spaces in Sections 14 to 16. The final Section considers obstructions to representing a (p,q)-link homotopy class (with $q \leq m-2$) by a map which embeds S^q. This is related to work of Kirk and Koschorke. Appendix A gives a complete treatment of the cases with $m \leq 3$, and Appendix B proves a technical result needed in Section 17.

J.Hillman

Keywords : belt map; spherical link maps into Euclidean spaces; Hilton-Milnor- theorem; unstable homotopy groups of wedges of spheres; link homotopy of (p,q)-link maps; link homotopy classes of (p,q)-link maps into R^m; difference map; bordism of framed submanifolds of Euclidean spaces

Classification :

∗57Q45 Knots and links in high dimensions (PL-topology)
55Q20 Homotopy groups of wedges, etc.
57M25 Knots and links in the 3-sphere
55Q10 Stable homotopy groups